AP Physics 1
Chapter 5:6-9: Gravitation

Newton’s Law of Universal Gravitation

\[ F_G = \frac{G m_1 m_2}{r^2} \]

\[ G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2} \]

\[ \frac{(kg \cdot m^2)}{kg \cdot m^2} \]

Example: An apple of mass 102 grams rests on the surface of the earth in an apple orchard. Calculate the gravitational force exerted on the apple by the earth.

\[ F = \frac{G m_A m_E}{r_E^2} = \frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(0.102 \text{ kg})(5.98 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2} = 1.00 \text{ N} \]

Example: Calculate the weight of a 102 g apple.

\[ F_g = mg = (0.102 \text{ kg})(9.80 \frac{m}{s^2}) = 1.00 \text{ N} \]

Newton’s Third Law. For an apple resting on the surface of the earth, if the earth exerts a force on the apple, then the apple exerts an equal and opposite force on the earth.

Why don’t we notice this second force?

Newton’s Second Law:

\[ \sum F = m a \]

\[ \begin{align*}
\text{apple} & : \quad \frac{\sum F}{m} = \frac{1.00 \text{ N}}{0.102 \text{ kg}} = 9.80 \frac{m}{s^2} \\
\text{earth} & : \quad \frac{1.00 \text{ N}}{5.98 \times 10^{24} \text{ kg}} = 1.67 \times 10^{-25} \frac{m}{s^2} \quad \text{tiny}
\end{align*} \]

We have always used \( g = 9.8 \text{ m/s}^2 \) for the acceleration due to gravity, but this is only the value close to the surface of the earth.

Using the two expressions we have for gravitational force,

\[ F_g = mg \]

\[ F_g = \frac{G m_A m_E}{r^2} \]

\[ mg = \frac{G m_A m_E}{r^2} \]

\[ g = \frac{GM_{\text{planet}}}{r^2} \]

It is more precise to say that \( g \) is the gravitational field strength at a distance \( r \) from the center of the earth.

Gravitational field strength is a vector, pointing toward the center of the planet, and decreases as \( r^2 \) increases.

\[ g = \frac{F_g}{m} = \frac{9.8 \text{ N}}{kg} \]
example calculate the gravitational acceleration and the weight of a 65 kg astronaut on the ISS, 400 km above the surface of the earth.

\[ g_{\text{ISS}} = \frac{GM_E}{r^2} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.78 \times 10^6)^2} \]

\[ g_{\text{ISS}} = 8.68 \frac{m}{s^2} \]

\[ F_g = mg = (65 \times 8.68 \frac{m}{s^2}) = 564 N \]

example The fictional planet Barsoom has a mass \( \frac{1}{5} \) that of earth, and a radius \( \frac{1}{2} \) the radius of earth. Calculate gravitational acceleration on the surface of Barsoom.

\[ \frac{g_B}{g_E} = \frac{G(\frac{1}{5}M_E)}{\frac{1}{2}r_E^2} = \frac{\frac{1}{5}GME} {\frac{1}{2}E^2} = 4 \frac{1}{5} = \frac{4}{5} \]

\[ g_B = \frac{4}{5} g_E = 7.84 \frac{m}{s^2} \]

Satellites

Once a satellite is lifted to height at which it will orbit, it is given tangential speed (no friction, so it will continue at this speed)

Earth's gravity pulls it down below its expected path and into a circular path.

The satellite is falling around the earth.

Satellite Equation

The gravitational force acting on a satellite provides the radial acceleration:

\[ \frac{GM_E}{r^2} = \frac{m}{r^2} \]

\[ a_r = \frac{v^2}{r} \]

\[ v^2 = \frac{GM_E}{r} \]

orbital speed of a satellite

\[ v = \sqrt{\frac{GM_{\text{planet}}}{r}} \]

to put cannonball in orbit

\[ v = 7910 \frac{m}{s} \]

(7.9 km/s)
example 5-14 At what height above the earth’s surface must a geosynchronous satellite orbit? \( T = 24 \text{ hr} = 86,400 \text{s} \) stays above same point on earth

\[
\begin{align*}
V &= \frac{2\pi r}{T} \quad \text{and} \quad V = \sqrt{\frac{GM_e}{r}} \\
\left(\frac{2\pi r}{T}\right)^2 &= \left(\frac{GM_e}{r}\right)^2 \\
\frac{4\pi^2 r^3}{T^2} &= \frac{GM_e T^2}{r^3} \\
4\pi^2 r^3 &= GM_e T^2 \\
r &= \frac{3\sqrt{GM_e T^2}}{4\pi^2} \\
r &= \frac{3\sqrt{(6.7\times10^{-11})(5.97\times10^{24})(86,400)^2}}{4\pi^2} \\
r &= 4.22 \times 10^7 \text{ m} \\
- 6.38 \times 10^6 \text{ m} &\text{ subtract radius} \\
\frac{3.59 \times 10^7 \text{ m}}{3.59 \times 10^7 \text{ m}} &\text{ altitude}
\end{align*}
\]

Kepler’s Laws
1. Planet’s orbits are ellipses, sun at one focal point.
2. Planets sweep out equal areas in equal amounts of time
3. Comparing the period \( T \) to the mean distance from the sun to a planet \( r \)

\[
\frac{T^2}{R^3} = \text{constant}
\]

Derive Kepler’s 3rd Law for a circular orbit:

\[
\begin{align*}
\text{gravitational force causes circular (m) motion of a planet orbiting sun (Ms).} \\
\text{compute 2 planets} \\
\left(\frac{T^2}{R^3}\right)_{\text{planet 1}} = \left(\frac{T^2}{R^3}\right)_{\text{planet 2}} \\
\frac{GM_s M_p}{R^2} &= \frac{m v^2}{R} = \frac{4\pi^2 R}{T^2} \\
GM_s T^2 &= 4\pi^2 R^3 \\
\frac{T^2}{R^3} &= \frac{1}{GM_s}
\end{align*}
\]
AP Physics 1 Problems: Gravitation
1. What is the force of gravity between the Earth and the Sun?
2. What is the force of gravity between a 10 kg mass on the surface of the Earth, and the Earth?
3. The force of gravity between two asteroids is 100 newtons. What will be the force of gravity if:
   (a) The mass of one of the satellites doubles?
   (b) The distance between the satellites is 1/2 as large?
4. An object has a weight of 40 Newton’s on the surface of the Earth. What will be its weight if it is moved to a distance of 1 Earth’s radii above the surface of the Earth? (You may want to sketch this one!)
5. What is the force of gravity between two protons (m_p = 1.67 x 10^{-27} kg) which are 1 x 10^{10} meters apart? What is the acceleration of one of the protons?
6. What is the force of gravity between the Earth and a 200 kg satellite located 500,000 m above the surface of the Earth?
7. The force of gravity between an asteroid and a planet is F.
   (a) What will be the force of gravity if the mass of the planet doubles?
   (b) What will be the force of gravity if the distance between them triples?
   (c) What will be the force of gravity if the mass of both doubles, while the distance between them halves?
8. NASA wants to place a 200 kg satellite in orbit around the Earth, at a distance of 6.9 x 10^{6} meters from the center of the Earth.
   (a) What is the force of gravity on the satellite?
   (b) What is the acceleration of the satellite?
   (c) What must be the velocity of the satellite?
   (d) How long will it take the satellite to orbit the Earth once?
9. What must be the speed of a satellite which is to orbit the Earth at a distance of 4 x 10^{7} meters from the center of the Earth? How long will it take to orbit the Earth once?
10. A planet with a mass of 4 x 10^{24} kg orbits a star of mass m, at a distance of 4 x 10^{15} meters once every 450 days.
    (a) What is the time it takes the planet to orbit in seconds?
    (b) What is the speed of the planet?
    (c) What is the acceleration of the planet?
    (d) What is the force on the planet?
    (e) What is the mass of the star?
11. At what altitude above the surface of the earth would the acceleration due to gravity be 5.0 m/s^2?
12. How fast would you have to throw a ball to put it in low earth orbit? Neglect air resistance and mountains.
13. A satellite in a circular orbit just above the surface of the moon.
    (a) What is the acceleration of the satellite?
    (b) What is the speed of the satellite?
    (c) What is the period of the satellite in its orbit?

Book Problems p. 131 Problems 28, 29, 30, 31, 32, 41, 48, 49, 65