6.3(a)  Vectors in the Plane

Goals:
Students will be able to . . .

• represent vectors as directed line segments.
• write the component forms of vectors.
• perform basic vector operations and represent vectors graphically.
• write vectors as linear combinations of unit vectors.
• find the direction angles of vectors.
• use vectors to model and solve real-life problems.

A vector is a directed line segment.

Naming a Vector
Initial point then terminal point
\( \overrightarrow{PQ} \)  
(vector PQ)

Vectors named using lowercase, boldface letters \((v)\).  \(\vec{v}\)

\( \|\overrightarrow{PQ}\| = \text{magnitude of vector PQ} \)

Magnitude is found using the Distance Formula.

Two directed line segments that have the same magnitude and direction (slope) are equivalent.
Example #1

Let \( \mathbf{u} \) be represented by the directed line segment from \( P = (0, 0) \) to \( Q = (3, 1) \), and let \( \mathbf{v} \) be represented by the directed line segment from \( R = (2, 2) \) to \( S = (5, 3) \). Show that \( \mathbf{u} = \mathbf{v} \).

(a) Find the magnitude of each vector.
\[
\| \mathbf{u} \| = \sqrt{(3-0)^2 + (1-0)^2} = \sqrt{10}
\]
\[
\| \mathbf{v} \| = \sqrt{(5-2)^2 + (3-2)^2} = \sqrt{10}
\]

(b) Find the slope of each vector.
\[
\mathbf{u} \text{ m} = \frac{1 - 0}{3 - 0} = \frac{1}{3}
\]
\[
\mathbf{v} \text{ m} = \frac{3 - 2}{5 - 2} = \frac{1}{3}
\]

(c) Interpret the information.
The vectors \( \mathbf{u} \) and \( \mathbf{v} \) are equivalent, because they have the same magnitude and direction.
Component form of a vector is a notation identifying the horizontal and vertical changes in a vector from the initial point to the terminal point.

Horizontal component is ...
- positive for movement to the right
- negative for movement to the left

Vertical component is ...
- positive for movement upwards
- negative for movement downwards

Notation for the component form of a vector

Use "pointed brackets"

Horizontal component is listed first, vertical component is second

\[ <\text{horizontal component, vertical component}> \]
To find the component form of a vector (without a graph)

Terminal coordinates \((x_t, y_t)\)

Initial coordinates \((x_i, y_i)\)

\(<x\text{-component, } y\text{-component}>\)

\(<x_t - x_i, y_t - y_i>\)

Magnitude of a vector = \(\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}\)

(coordinates)

Magnitude of a vector = \(\sqrt{(x\text{-component})^2 + (y\text{-component})^2}\)

(component form)

\(\vec{u} = <3, -5>\)

\(||\vec{u}|| = \sqrt{(3)^2 + (-5)^2}\)

\(||\vec{u}|| = \sqrt{9 + 25}\)

\(||\vec{u}|| = \sqrt{34}\)
Example #2

Find the component form and magnitude of the vector \( \mathbf{v} \) starting at \((-4, 3)\) and terminating at \((8, -7)\).

\[
\mathbf{v} = \langle 12, -10 \rangle \\
\|\mathbf{v}\| = \sqrt{144 + 100} = 2\sqrt{61}
\]

Practice

Find the component form and the magnitude of each vector \( \mathbf{v} \).

(a) Initial Point \((-3, 11)\) Terminal Point \((9, 40)\)

\[
\mathbf{v} = \langle 12, 29 \rangle \\
\|\mathbf{v}\| = \sqrt{144 + 841} = \sqrt{985}
\]

(b) Initial Point \((\frac{7}{2}, 0)\) Terminal Point \((0, -\frac{7}{2})\)

\[
\mathbf{v} = \langle -\frac{7}{2}, -\frac{7}{2} \rangle \\
\|\mathbf{v}\| = \frac{\sqrt{49 + 49}}{2} = \frac{7\sqrt{2}}{2}
\]

(c) Initial Point \((\frac{5}{2}, -2)\) Terminal Point \((1, \frac{2}{5})\)

\[
\mathbf{v} = \langle -\frac{3}{2}, \frac{12}{5} \rangle \\
\|\mathbf{v}\| = \sqrt{\frac{801}{100} - 3 \cdot \frac{89}{10}} = \sqrt{\frac{801 - 3 \cdot 89}{10}} = \sqrt{\frac{801 - 267}{10}} = \sqrt{\frac{534}{10}} = \sqrt{53.4}
\]

P429: 13–23(odd)