1.4 Continuity and The Intermediate Value Theorem

THEOREM 1.13 Intermediate Value Theorem
If $f$ is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and $k$ is any number between $f(a)$ and $f(b)$, then there is at least one number $c$ in $[a, b]$ such that $f(c) = k$.

1) Make sure $f$ is continuous on the interval
2) Find $f(a)$
3) Find $f(b)$
4) $c$ should be between $a$ and $b$
5) $f(c)$ must be between $f(a)$ and $f(b)$

In 1-4, explain why the function has a zero in the given interval.

1) $f(x) = \frac{1}{16}x^4 - x^3 + 3$, $[1, 2]$
   $f(1) = \frac{1}{16} - 1 + 3 > 0$ pos.
   $f(2) = \frac{1}{16}(2)^4 - (2)^3 + 3 = 1 - 8 + 3 < 0$ neg.
   $x = 1.4900$

In 5-8, verify that the Intermediate Value Theorem guarantees that there is a zero in the interval $[0, 1]$ for the given function. Use a graphing calculator to find the zero.

5) $f(x) = x^3 + x - 1$
   $0, 0.8, 2.3$

In 9-12, verify that the Intermediate Value Theorem applies to the indicated interval and find the value guaranteed by the theorem. No calculator is permitted on these problems.

9) $f(x) = x^2 + x - 1$, $[0, 5]$.
   $f(0) = -1$
   $f(5) = 25 + 5 - 1 = 29$
   $x^2 + x - 1 = 11$
   $x^2 + x - 12 = 0$
   $(x + 4)(x - 3) = 0$
   $x = -4, 3$

15) Show that $f(x)$ is continuous at $x = 2$ for $f(x) = \begin{cases} 5 - x, & -1 \leq x \leq 2 \\ x^2 - 1, & 2 < x \leq 3 \end{cases}$
   $5 - 2 = x^2 - 1$
   $3 = 3$

Continuity at a Point - You must prove 3 things.
1) $f(c)$ is defined
2) $\lim_{x \to c} f(x)$ exists
3) $\lim_{x \to c} f(x) = f(c)$